Final exam Calculus-3 (10 points for free)

<u>Part A</u>: G. Palasantzas (Problems 1-3)

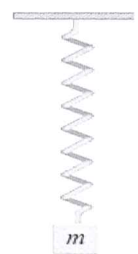
Problem 1 (5 points) Prove that if $a_n \geqslant 0$ and $\Sigma\, a_n$ converges, then $\Sigma\, a_n^2$ also converges.

Problem 2 (20 points) (a: 10 points) Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{n}{b^n}(x-a)^n, \quad b>0$

(b: 10 points) Consider the sequence $f_n(x)=\frac{\cos(nx)}{n^4}$ with $x \in (-\infty, +\infty)$

Is $f_n(x)$ uniformly convergent ?

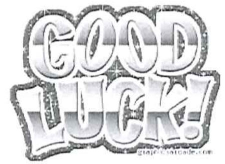
Problem 3 (20 points) Suppose a mass m is attached to a spring with spring constant k, and let $k = m\omega^2$. If an external force $F(t) = F_o \cos(\omega t)$ is applied, then we have:



Equation of motion: → $$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

If we assume $c^2 - 4mk < 0$, then show that the motion is described by:

$$x(t) = e^{-(c/2m)t}\left[c_1\cos(\tilde{\omega}t) + c_2\sin(\tilde{\omega}t)\right] + \left(\frac{F_o}{c\omega}\right)\sin(\omega t), \quad \text{with} \quad \tilde{\omega} = \omega\sqrt{1-(c/2m\omega)^2}$$



<u>Part B</u>: P.D. Barthel (Problems 4-6)

Problem 4 Consider the steady-state situation of a heat conducting square plate with surface area 1, of which three sides have temperature zero and one side has temperature 5. Give the Laplace equation of this situation and solve that equation. (15 points).

Problem 5 Expand $f(x) = x$ on the interval $0 < x < 2$, in a (half-range) sine series, and subsequently infer from that series an expansion for the number π. $(10 + 5$ points).

Problem 6 Discuss and illustrate the methods of Fourier *integrals* for finding the solution of a partial differential equation for one or two of the examples given in the lectures, by working out the global solution (15 points).